

Modeling and Validating Hybrid Systems Using VDM and Mathematica

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Abstract

Hybrid systems are characterized by the hybrid evolution of their state: A part of the state changes discretely, the other part changes continuously over time. Typically, modern control applications belong to this class of systems, where a digital controller interacts with a physical environment. In this article we illustrate how a combination of the formal method VDM and the computer algebra system Mathematica can be used to model and simulate both aspects: the control logic and the physics involved. A new Mathematica package emulating VDM-SL has been developed that allows the integration of differential equation systems into formal specifications. The SAFER example from [11] serves to demonstrate the new simulation capabilities Mathematica adds: After the thruster selection process, the astronaut's actual position and velocity is calculated by numerically solving Euler's and Newton's equations for rotation and translation. Furthermore, interactive validation is supported by a graphical user interface and data animation.

1 Introduction

Modern control applications are realized through microcontrollers executing rather complex control logics. This complexity is increased by the fact that control software interacts with a physical environment through actors and sensors. Such systems are called *hybrid systems* due to the hybrid evolution of their state: One part of the state (variables) changes discretely, the other part changes continuously over time.

Hybrid systems are excellent examples for motivating the use of formal software development methods. First, their complexity calls for a real software engineering discipline applying both, a pro-

cess model as well as a mathematical method. Second, these kinds of systems are often safety-critical which justifies formal validation and verification techniques. Third, engineers in the control domain are educated in the use of mathematical models for designing dynamic systems.¹ In our experience, the offer of a formal method for software development is more often appreciated by control engineers, than by software developers used to produce short cycle products in 'Internet time'.

In [11] the hybrid system SAFER has been chosen by NASA in order to introduce to formal specification and verification techniques. SAFER is an acronym for "Simplified Aid For EVA (Extravehicular Activity) Rescue". It is a small, lightweight propulsive backpack system designed to provide self-rescue capabilities to a NASA space crewmember separated during an EVA. In this NASA guidebook[11], SAFER is specified formally in the PVS notation and properties are formally proved using the PVS theorem prover [12]. In the guidebook the dynamic aspects are used to compare the continuous domain model from spacecraft attitude control with the discrete PVS model of SAFER's control logic. It demonstrates that the two models have the same goals: rigorous description and prediction of behavior but that the needed mathematics and calculation techniques are different.

In [1, 2] Agerholm & Larsen have proposed a cheaper testing based validation approach to the SAFER example using an executable VDM-SL model and the IFAD VDM-SL Toolbox [10, 7, 6]. They recommend the use of a specification executor and animator for raising the confidence in a formal model prior to formal proving.

We agree with Agerholm & Larsen's arguments for such a "light-weight" approach to formal meth-

¹The same holds for software developers coming from classical engineering disciplines.

ods in order to facilitate the technology transfer. Since in several industrial projects performed at our institute a similar experience has been made [9, 15, 5], one of our research areas has become the support of testing through formal methods [4].

However, neither the PVS nor the VDM-SL model of SAFER did take the continuous physical models into account. The reason is that, in general, today’s formal method tools are not well suited for supporting continuous mathematics. This paper shows a solution the problem.

In the following it is demonstrated how an explicit discrete model can be combined with the continuous physical model for validation and animation. With the right tool there is no reason why a physical model should not be included in the validation process of a hybrid system. Just the opposite is the case: [1] detected several cases where the interface to a cut out automatic attitude hold (AAH) control unit needed further clarification.

In this work the commercial computer algebra system Mathematica [16] has been used to overcome the gap between discrete and continuous mathematics. A VDM-SL package has been implemented that allows to specify in the style of the Vienna Development Method (VDM) inside Mathematica. Thus, explicit discrete models can be tested in combination with differential equation systems modeling physical behavior by solving the equations on the fly. Even pre- and post-condition checking is possible. Again, NASA’s SAFER system serves as the demonstrating example. The VDM-SL specification of [2] has been taken and extended with the physics involved in SAFER, expressed through differential equations. More precisely, the physical behavior is movement in space, modeled by the laws for translation and rotation — Newton’s and Euler’s equations for three dimensional space.

Beside the execution (testing) of hybrid models, Mathematica’s front-end supports the visual validation of such systems. The graphical user-interface for SAFER’s hand grip is implemented inside the computer algebra system as well as a scientific graph representing the movement of a crew-member using SAFER. After each control cycle, actual physical vectors like angular velocity or acceleration can be inspected together with the logical status, e.g. the thrusters firing. Finally, it is even possible to animate a sequence of performed control-cycles as a movie showing the SAFER representation flying.

The structure of the rest of the paper is as follows. First in Section 2 an overview of the SAFER system is given, which will serve as the demonstrat-

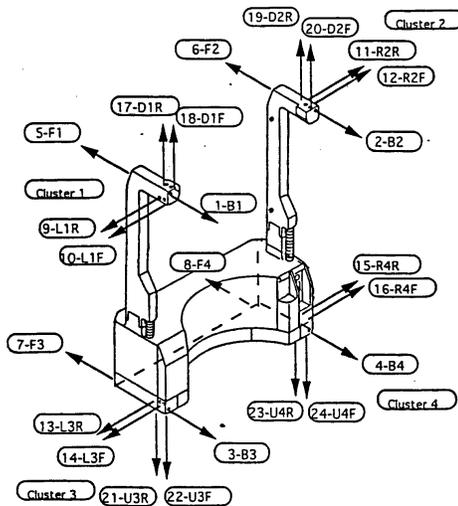


Figure 1. SAFER thrusters.

ing example throughout the paper. This is followed by a discussion of VDM-SL and its realization inside Mathematica in Section 3. Then, a description of the discrete SAFER model is given in Section 4. Section 5 explains the differential equation systems modeling SAFER’s physics and the coordinate transformations needed. Then, Section 6 introduces to the hybrid model and demonstrates the integration of VDM-SL and differential equation systems. Next, the validation capabilities of our approach are discussed in Section 7 and Section 8. In the final Section 9 we draw some conclusion regarding the presented work in particular, as well as possible future approaches in general.

2 The SAFER System

The following overview of the SAFER system is based on, and partly copied from, the NASA guidebook [11], which describes a cut-down version of a real SAFER system.

The Simplified Aid for EVA Rescue (SAFER) is a small, self-contained, backpack propulsion system enabling free-flying mobility for a NASA crewmember engaged in extravehicular activity (EVA). It is intended for self-rescuing on Space Shuttle missions, as well as during Space Station construction and operation, in case a crewmember got separated from the shuttle or station during an EVA. This type of contingency can arise if a safety tether breaks, or if it is not correctly fastened. SAFER attaches to the underside of the Extravehicular Mobility Unit

(EMU) primary life support subsystem backpack and is controlled by a single hand controller that is attached to the EMU display and control module. Figure 1 shows the backpack propulsion system with the 24 gaseous-nitrogen (GN_2) thrusters, four in each of the positive X , Y and Z directions. For example, the thrusters denoted by 5-F1, 6-F2, 7-F3 and 8-F4 are firing backwards (indicated by the arrows) resulting in a forward motion.

The main focus of the discrete specification is on the thruster selection logic, which is rather complex due to a required prioritization of hand controller commands. Various display units and switches which are not directly related to the selection of the thrusters have been ignored in our model. However, in contrast to [11] and [1] the calculation of the control output in the Automatic Attitude Hold (AAH) is not ignored, but simulated based on a dynamic model of the physics discussed in Section 5.

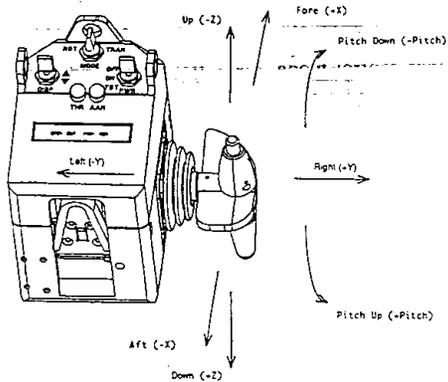


Figure 2. Hand controller module of SAFER.

The hand controller, shown in Figure 2, is a four-axis mechanism with three rotary axes and one transverse axis using a certain hand controller grip. A command is generated by moving the grip from the center null position to mechanical hard-stops on the hand controller axes. Commands are terminated by returning the grip to the center position. The hand controller can operate in two modes, selected via a switch, either in translation mode, where X (forward-backwards), Y (left-right), Z (up-down) and pitch commands are available, or in rotation mode, where roll, pitch, yaw and X commands are available. The arrows in Figure 2 show the rotation mode commands. Note that X and pitch commands are available in both modes.

Pitch commands are issued by twisting the hand grip around its transverse axis, while the other commands are obtained around the rotary axis.

A push-button switch on top of the grip initiates and terminates AAH according to a certain protocol. If the button is pushed down once the AAH is initiated, while the AAH is deactivated if the button is pushed twice within 0.5 seconds.

As mentioned above there are various priorities among commands that make the thruster selection logic rather complicated. Translational commands issued from the hand controller are prioritized, providing acceleration along a single translational axis, with the priority X first, Y second, and Z third. When rotation and translation commands are present simultaneously from the hand controller, rotations take higher priority and translations are suppressed. Moreover, rotational commands from the hand grip take priority over control output from the AAH, and the corresponding rotation axes of the AAH remain off until the AAH is reinitialized. However, if hand grip rotations are present at the time when the AAH is initiated, the corresponding hand controller axes are subsequently ignored, until the AAH is deactivated.

In [1] it is explained how a specification interpreter tool facilitates the validation of the requirements listed in the appendix of the NASA guidebook. Moreover, it is demonstrated that formal validation techniques uncover open issues in informal requirements even if they seem to be straightforward and clear.

The same validation techniques as discussed in [1] can be applied in our Mathematica based framework — and more. However, before we discuss the value added through a hybrid model, in the following section, the realization of our VDM-SL package is discussed.

3 VDM-SL in Mathematica

VDM-SL is the specification language of the Vienna Development Method (VDM) [10, 7]. VDM is a widely used formal method, and it can be applied to the construction of a large variety of systems. It is a model-oriented method, i.e. its formal descriptions (specifications) consist of an explicit model of the system being constructed. More precisely mathematical objects like sets, sequences and finite mappings (maps) are used to model a system's global state. Additional logic constraints, called data-invariants, allow one to model informal requirements by further restricting specified data-

types. For validation purposes the functionality may be specified explicitly in an executable subset of VDM-SL. In addition, pre- and post-conditions state *what* must hold before and after the evaluation of a system's operation. Although VDM-SL is called a general purpose specification language it does not support the specification of dynamic systems. The language's ISO-standard [13] does not even include standard functions like sine or cosine.

Here, as the name indicates, Mathematica's strengths supplement our combined approach. Mathematica is a symbolic algebra system that offers the opportunity of solving arbitrary non-linear as well as linear systems of equations. Mathematica's language interpreter is in fact a rewriting system providing an untyped functional programming language. For an introduction to functional programming in Mathematica see [3]. This programming language has been used in order to define a package emulating the specification language VDM-SL. By emulating we express the fact that the package does not allow one to write specifications in VDM-SL's concrete syntax, but in its abstract syntax with some pretty printing for VDM-SL output.

Mathematica's user interface are so called notebooks, fancy editors structured in cells for input, output or plain text. Entering a Mathematica expression in an input cell, the system tries to evaluate this input through a rewriting procedure based on pattern matching.

The following language constructs have been added to the standard language in order to import the VDM-SL model from [2]:

- abstract datatypes for composite types, sets, sequences and maps
- comprehension expressions for sets, sequences and maps
- let and cases expressions
- operators for propositional and predicate logic
- types optionally restricted by data-invariants
- value and global state definitions
- typed function/operation definitions with pre- and post-conditions

Some of the items above deserve a more detailed discussion.

Comprehensions

A powerful feature of a specification language like VDM-SL is its ability to construct collection types like sets, sequences and maps through comprehensions. For example, a set-comprehension defines a set through an arbitrary expression describing the set-elements with its free variables ranging over a set of values, such that an optional condition holds. The following example demonstrates the value added through a computer algebra system. The set-comprehension

$$\text{set}\{x|\{x \in \mathbb{Z}\} \cdot \{x^6 - 44x^5 + 318x^4 + 4102x^3 - 4461x^2 + 550x + 8750 == 0\}}$$

represents a set of elements x , where x is an integer number such that the equation holds.

The resulting set²

$$\text{set}[-7, -1, 25]$$

demonstrates that, unlike IFAD's VDM-SL interpreter, comprehensions ranging over infinite sets may be evaluated.

Types

As already mentioned, in contrast to VDM, Mathematica has an untyped language. Consequently, no type checking mechanism is available. However, types are an important tool for specifying a data-model in VDM. Therefore, type declarations of the form `Type[name, type]` have been included, where `type` is one of the predefined VDM-SL types, like basic types, composite types, sets ... For example, a type `ISet` representing a set of natural numbers might be declared by `Type[ISet, set[N]]`.

Optionally, a type can be further constrained by a data-invariant condition. Such invariant types are defined by `Type[name, type, Invariant - > predicate]`. The `predicate` is defined by a lambda expression mapping `type` to a Boolean value. All the invariants are globally stored in the system for invariant checking, before and after the evaluation of a VDM function.

Internally, a type is translated to a Mathematica pattern, matching those values the type denotes. Invariant types are supported by the possibility of defining patterns with arbitrary predicates. These patterns restrict the argument range in the definition of typed VDM functions.

²The six solutions including double and complex solutions are: $-7, -1, 1 - I, 1 + I, 25, 25$.

Functions

Using the VDM-SL package, typed functions with pre- and post-conditions can be defined using the constructor

```
VDMFunction[id, sig, id[vars] := body, pre, post]
```

with the following parameters:

`id` the name of the function,

`sig` the signature of the function,

`id[vars] := body` the function definition,

`pre` an optional pre-condition stating what must hold before the evaluation such that the post-condition holds,

`post` an optional post-condition stating what must hold after the evaluation.

VDMFunction realizes a complex call to Mathematica's internal Function call and emulates the checks for

- the signature types,
- pre- and post-condition,
- data-invariants.

4 Discrete Model

In order to demonstrate the Mathematica package the same functions for the thruster selection logic as in [1] are presented in this section. The six degree-of-freedom of the translation and rotation commands is modeled using a composite type:

```
Type[SixDofCommand, Composite[{"tran", TranCommand}, {"rot", RotCommand}]]
```

whose two fields are finite maps from translation and rotation axis respectively to axis commands. For example the type of translation commands is defined as follows:

```
Type[TranCommand, TranAxis -> AxisCommand, Invariant -> (dom[#] == set[X,Y,Z]&)]
```

where the invariant ensures that command maps are total. Here, the invariant predicate is defined by a lambda expression in Mathematica's notation of pure functions. The type of rotation commands is defined similarly. Enumerated types are used for axis commands and translation and rotation axes:

```
VDMFunction[
  SelectedThrusters,
  AUX'SixDofCommand × AUX'RotCommand ×
  set[AUX'RotAxis] × set[AUX'RotAxis]
  -> ThrusterSet,
  SelectedThrusters[hcm, aah, actAxes, ignHcm] :=
  let[{tran, rot, bfMandatory, bfOptional,
  lrudMandatory, lrudOptional, bfThr, lrudThr},
  {tran, rot} =
    (IntegratedCommands[hcm, aah, actAxes, ignHcm]
    /. SixDofCommand[tr_, ro_] :-> tr, ro);
  {bfMandatory, bfOptional} = BFThrusters[tran[X],
  rot[PITCH],
  rot[YAW]];
  {lrudMandatory, lrudOptional} =
    LRUDThrusters[tran[Y],
  tran[Z],
  rot[ROLL]];

  bfThr = If[(rot[ROLL] === ZERO),
  bfOptional ∪ bfMandatory,
  bfMandatory ];
  lrudThr = If[(rot[PITCH] === ZERO) and
  (rot[YAW] === ZERO),
  lrudOptional ∪ lrudMandatory,
  lrudMandatory];
  set @@ (bfThr ∪ lrudThr)
]
];
```

Figure 3. The SelectedThrusters function.

```
Type[AxisCommand, NEG | ZERO | POS];
```

```
Type[TranAxis, X | Y | Z];
```

```
Type[RotAxis, ROLL | PITCH | YAW]
```

In the `SelectedThrusters` function in Figure 3 grip commands from the hand controller (with six-degree-of freedom commands) are integrated with the AAH control output. The `IntegratedCommands` function prioritizes hand controller and AAH commands.

Based on these commands, thrusters for back and forward accelerations and left, right, up and down accelerations are calculated by two separate functions. Figure 4 presents cut-down versions of these functions. These represent a kind of look-up tables, modeled using cases expressions. Note that they return two sets of thruster names, representing mandatory and optional settings respectively.

5 Physics Involved in SAFER

This section presents the continuous model of the physics involved in our hybrid model. For the SAFER example, translation and rotation equations from mechanics are sufficient for modeling the motion of a crewmember using the propulsion system. The purpose of this model is twofold: First, we need to calculate the sensor inputs of angular velocity for simulating the AAH. Second, in order to visualize

```

VDMFunction[
  BFThrusters,
  AUX'AxisCommand × AUX'AxisCommand × AUX'AxisCommand
  -> ThrusterSet × ThrusterSet,
  BFThrusters[A, B, C] :=
  cases[{A, B, C},
    {NEG, ZERO, ZERO} -> {{B4}, {B2,B3}},
    {ZERO, ZERO, ZERO} -> {{}, {}},
    {POS, NEG, ZERO} -> {{F1,F2}, {}},
    ...
  ]
];

VDMFunction[
  LRUDThrusters,
  AUX'AxisCommand × AUX'AxisCommand × AUX'AxisCommand
  -> ThrusterSet × ThrusterSet,
  LRUDThrusters[A, B, C] :=
  cases[{A, B, C},
    {NEG, NEG, ZERO} -> {{}, {}},
    {NEG, ZERO, ZERO} -> {{L1R,L3R}, {L1F,L3F}},
    {POS, ZERO, POS} -> {{R2R}, {R2F,R4F}},
    ...
  ]
];

```

Figure 4. Extracts from BFThrusters and LRUDThrusters.

the SAFER movement, absolute coordinates have to be determined. The mathematics needed can be found in the standard literature of mechanics, like [8].

Translation

The translation of a crewmember wearing SAFER is described by Newton's second law of motion expressed by

$$F = m\dot{v} = \dot{p} \quad (1)$$

where F , m , v and p denote force vector, mass, velocity vector and impulse vector. It states that "The time rate of change of the momentum of a particle is proportional to the force applied to the particle and in the direction of the force."

Rotation

The rotation is modeled by three equations known as the *Euler's equations* of motion for the rotation of a rigid body.

Denote by Ω the angular velocity defined with respect to the center of mass, and by I the moments of inertia. The equations describing the body rota-

tions are then given by

$$I_1 \dot{\Omega}_1 + (I_3 - I_2) \Omega_2 \Omega_3 = Q_1 \quad (2)$$

$$I_2 \dot{\Omega}_2 + (I_1 - I_3) \Omega_3 \Omega_1 = Q_2 \quad (3)$$

$$I_3 \dot{\Omega}_3 + (I_2 - I_1) \Omega_1 \Omega_2 = Q_3 \quad (4)$$

or as a vector equation where I is a diagonal matrix:

$$I \cdot \dot{\Omega} + \Omega \times I \cdot \Omega = Q \quad (5)$$

Q_i denotes a torque causing a rotation around the i -axis, in the body's own coordinate system. Here, the torque is given by the sum over the thrusters firing. Actually, a component Q^{th} is calculated by the cross product of a thruster's position vector relative to the center of mass and its force. SAFER does not use proportional gas jets, but thrusters whose valves are open or not, which simplifies the calculation.

Motion

In order to combine translation and rotation in a single model of motion, suitable for our purposes, coordinate transformations are necessary. More precisely, the fixed coordinate system values for visualization (position and velocity) have to be related to SAFER's coordinate system values (angular velocity).

As Ω is calculated in the body's own coordinate system, they have to be transformed back to the fixed coordinate system. Given the Euler angles φ , θ and ψ that denote the deviation of the fixed x , y and z axis, the angular velocities can be calculated according to the following formula.

$$\Omega_1 = \dot{\varphi} \sin \theta \sin \psi + \dot{\theta} \cos \psi \quad (6)$$

$$\Omega_2 = \dot{\varphi} \sin \theta \cos \psi - \dot{\theta} \sin \psi \quad (7)$$

$$\Omega_3 = \dot{\varphi} \cos \theta + \dot{\psi} \quad (8)$$

The derivation of these equations can be found in [8]. Using vector notation we get the equation:

$$\Omega = D_3(\psi) \cdot D_1(\theta) \cdot (\dot{\theta}, 0, \dot{\varphi})^T + (0, 0, \dot{\psi})^T \quad (9)$$

$$D_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & -\sin \theta & \cos \theta \end{pmatrix} \quad (10)$$

$$D_3 = \begin{pmatrix} \cos \psi & \sin \psi & 0 \\ -\sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (11)$$

where D_1 and D_3 are rotation matrices that turn the coordinate system by a given angle.

D_1 and D_3 are used to transform a vector from our fixed coordinate system to a turned coordinate system. For translation motion, the thruster's force vector F has to be transformed from SAFER's coordinate system to the fixed one using the transposed rotation matrices:

$$(D_3(\psi) \cdot D_1(\theta) \cdot D_3(\varphi))^T$$

Summarizing, these four vector differential equations are sufficient for modeling SAFER's motion over time:

$$v = \dot{x} \quad (12)$$

$$m \cdot v = (D_3(\psi) \cdot D_1(\theta) \cdot D_3(\varphi))^T F \quad (13)$$

$$I \cdot \dot{\Omega} + \Omega \times I \cdot \Omega = Q \quad (14)$$

$$\Omega = D_3(\psi) \cdot D_1(\theta) \cdot (\dot{\theta}, 0, \dot{\varphi})^T + (0, 0, \dot{\psi})^T \quad (15)$$

Solving these equations with given thruster forces results in SAFER's position vector $x(t)$ and the angular velocity $\Omega(t)$ used for AAH.

Alternatives to the Euler's equations model are possible. For example, an approach could have involved the less computationally intensive quaternions. However, for validation purposes the model should be as intuitive as possible, here efficiency plays a minor role.

6 A Hybrid Model

The hybrid model of SAFER consists of the hand controller and the Automatic Attitude Hold as its discrete parts on one side and the equations of motion as the continuous part on the other side. Both are modeled in Mathematica, the first in the form of the VDM-SL specification using our VDM-SL emulation package, the later in the form of ordinary differential equations in Mathematica notation.

The combination of the discrete control system and the continuous physical model during the testing phase carries certain advantages:

Not only can the system specification be tested in an (idealized) physical simulation, but also the system parameters like the force of the thrusters and the moments of inertia of the backpack can easily be adjusted until the system responds in a way suitable for practical use.

This is not a very rigorous approach, and it is not intended to replace other testing tools and methods. Rather it can serve as a valuable supplementary tool.

```
VDMFunction[
ControlCycle,
SwitchPositions × HandGripPosition ×
RotCommand × InertialRefSensors -> ThrusterSet,

ControlCycle[SwitchPositions[mode_, aah_], rawGrip,
aahCmd, IRUSensors]:=
let[{{
gripCmd=HCM'GripCommand[rawGrip, mode],
thrusters=SelectedThrusters[gripCmd, aahCmd,
AAH'ActiveAxes[], AAH'IgnoreHcm[]]
}},
AAH'Transition[IRUSensors, aah, gripCmd, SAFER'clock];
SAFER'clock=SAFER'clock+1;
PosData=CalcNewPosition[thrusters];
thrusters
],
True,
card[RESULT] ≤ 4 ∧ ThrusterConsistency[RESULT]
];

VDMFunction[
SensorControlCycle,
SwitchPositions × HandGripPosition -> ThrusterSet,

SensorControlCycle[SwitchPositions[mode_, aah_],
rawGrip]:=
ControlCycle[SwitchPositions[mode,aah],rawGrip,
AAHControlOut[Sensors], Sensors ]
];
```

Figure 5. The ControlCycle function.

The Control Cycle

The ControlCycle function (Figure 5) integrates the discrete model of hand control, thruster selection and Automatic Attitude Hold (AAH) with the continuous physical model of motion presented above.

The Control Cycle is implemented in two different functions. ControlCycle takes the state of the hand control (switches and hand grip) as well as the already calculated or manually entered AAH commands and the sensor values. SensorControlCycle takes the values of the sensors (here simulated by the solutions of the equations of motion of the previous control cycle) and determines which thrusters are invoked by the AAH. These are then passed on to ControlCycle.

After determining the active thrusters and the AAH state, the differential equations are solved numerically in the CalcNewPosition function and the current position is updated. These results simulate the values measured by the sensors (with exception of the heat sensors, which are left out in our model) providing data for AAH. This part of the control system is completely left out in [1] and only included in the form of two unspecified functions in the PVS model [11].

Here the SAFER state is not as trivial as in [1] where it holds only a clock variable.

```

VDMFunction[
  AAHControlOut,
  InertialRefSensors->RotCommand,

  AAHControlOut[IRUSensors]:=
  let[{{rr=IRUSensors."RollRate",
        pr=IRUSensors."PitchRate",
        yr=IRUSensors."YawRate"}},
    map[
      ROLL->Which[
        rr <= -EpsRoll, POS,
        rr >= EpsRoll, NEG,
        True, ZERO],
      ...
    ]
  ];

```

Figure 6. The Bang Bang algorithm for AAH.

```

State[SAFER,
  Type[clock, N],
  Type[PosData, PositionData],
  Type[Sensors, InertialRefSensors],
  Type[step, Rpos],
  Type[PosDataList, List[PositionData]],

  init[SAFER] := SAFER[0,
    PositionData[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
    InertialRefSensors[0, 0, 0, 0, 0, 0, 0, 0, 0],
    1/4, {{{0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}}}]
];

```

The state above also includes the current position, Euler angles and velocities stored in a variable of type `PositionData`.

Even the past position data is stored for providing full information about SAFER's trajectory. For simulation this data will be used to display the history as a Mathematica "movie" showing the astronaut flying around in the coordinate system.

Automatic Attitude Hold (AAH)

Simulating the measured sensor values by the results of the equations of motion provides the opportunity of including the Automatic Attitude Hold mechanism by a simple Bang Bang [11] algorithm: If the angular velocity for an axis where AAH is turned on exceeds a certain threshold, selected thrusters are fired in order to slow down this rotation (Figure 6). AAH is limited to this mechanism because SAFER is only based on simple thrusters with two states: on and off.

The Differential Equations

The equations of motion used to determine the new position of the astronaut are Newton's and Euler's equations described above. Although this

model neglects any gravitational forces and other disturbing influences, they could easily be added by an additional acceleration in the equations or random fluctuations applied to the results of the differential equations.

The new position is obtained by numerically solving the equations rather than algebraically which is less time-efficient, beside the fact that the algebraic solution is not necessary as only the result at time `step` is needed for simulation.

Since the equations are only slightly coupled, they can be solved in four steps, which is numerically more stable than solving them all at once. This functionality is provided by Mathematica's `NDSolve` function, which takes the differential equations and the initial conditions and returns numeric functions that approximate the exact solutions of the equations. In this case the trajectory is calculated piecewise: in every control cycle the trajectory only for that cycle is solved using the position before the cycle as the initial conditions and the force and torque applied by the thrusters as parameters. These can easily be calculated, the force by a simple vector addition of the forces applied by every single thruster, and the torque by adding up the cross products of the thruster positions with the force applied by that thruster.

First, Euler's equation in the astronaut's coordinate system is solved giving the angular velocity. This needs the forces and the torque applied by the fired thrusters as parameters. The result is then transformed back to the fixed coordinate system and used to solve the differential equation for the Euler angles. In a third step Newton's equation can be solved using the results from the previous equations. Finally, a simple integration of the velocities gives the position of the astronaut.

These numerical solutions to the differential equations can also be used to investigate stability. In the simplified case without any external forces like gravitation, this might not be so interesting, but as soon as external forces are modeled into the differential equations, stability is a crucial concern. What happens if the hand controller keeps in the same position over a long period of time? Such questions can easily be answered by solving the differential equations for a time period longer than just the control cycle.

7 Simulating SAFER

Mathematica does not only provide algebraic and numeric functionality, but also an extensive reper-

toire of plotting functions. Thus Mathematica has also been used to visualize SAFER’s current position together with other state information.

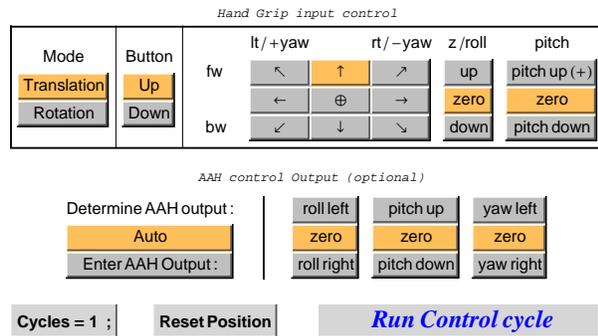


Figure 7. The GUI for the hand controller.

An interface to the hand controller similar to that in [2] is provided in Mathematica (Figure 7). It contains buttons for all the hand controller states as well as for manual input of the AAH output for overriding the simulated AAH in the model.

Pressing one of the buttons sets a global variable that is used to determine the parameters passed to the `ControlCycle` function. Additionally, the "Cycles=1" button determines how many control cycles should be evaluated when the "Run Control Cycle" button is pressed.

Pressing "Run Control Cycle" initiates the control cycle and after calculating the new position prints out a plot of the astronaut’s path so far together with his orientation indicated by the axes of his own coordinate system (Figure 8). Additionally, his velocity and angular velocity are shown as vectors. Optionally a table with the list of the fired thrusters as well as the axes where AAH is turned on is printed.

Since all the previous position data is stored, Mathematica can even animate this graph so that one can inspect the SAFER moving through space.

A graphical interface to the simulation like in Figure 7 is interesting when testing the system’s behavior in general. However, when adjusting parameters or testing specific cases, it’s more convenient to run the control cycles directly using Mathematica input commands. Figure 9 shows the input to create Figure 8.

In [1] the visualization is done outside the toolbox using dynamic link modules, which are programmed specifically for this one application. In Mathematica, changing only the differential equations suffices to include other influences like

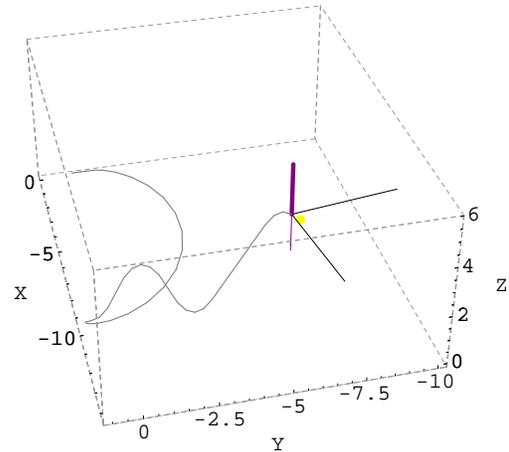


Figure 8. A sample trajectory of the SAFER.

gravity, as Mathematica chooses the algorithm to solve the equations.

However, testing in Mathematica is not restricted to graphical simulation. Like in [1], the output of the thruster selection logic can be validated by enumerating all possible states of the Hand controller, or in an extended version enumerating all possible states of the hand controller and the AAH. Figure 10 shows these functions formulated in Mathematica’s VDM-SL notation. On every possible state, `ControlCycle` is applied to calculate the fired thrusters. The result of this large map comprehension then has to be investigated manually.

Another important part in the process of verifying software would be coverage testing, which is unfortunately not possible in Mathematica.

8 Enhanced Analysis of the System

The simulation possibilities described in the last section can be exploited for risk and safety analysis of the system. A very simple application is the case when one of the thrusters fails due to a mechanical defect or an iced valve. The most important questions in this scenario are whether the astronaut will still be able to navigate the system, and whether it is possible to return before the air or the nitrogen for the thrusters is used up.

We investigated the functionality of AAH in the case where one thruster (6-F2) fails. Figure 11 shows the angular velocity of the system, with the

```

ResetSAFERPosition[ ];
(* 1 right *)
Do[SensorControlCycle[SwitchPositions[TRAN,UP],
  HandGripPosition[ZERO,ZERO,POS,ZERO]],{1}];
(* 3 yaw *)
Do[SensorControlCycle[SwitchPositions[ROT,UP],
  HandGripPosition[ZERO,ZERO,POS,ZERO]],{3}];
(* 15 "right" *)
Do[SensorControlCycle[SwitchPositions[TRAN,UP],
  HandGripPosition[ZERO,ZERO,POS,ZERO]],{15}];
(* wait *)
Do[SensorControlCycle[SwitchPositions[TRAN,UP],
  HandGripPosition[ZERO,ZERO,POS,ZERO]],{2}];
(* 3 up *)
Do[SensorControlCycle[SwitchPositions[TRAN,UP],
  HandGripPosition[POS,ZERO,ZERO,ZERO]],{3}];
(* 6 down *)
Do[SensorControlCycle[SwitchPositions[TRAN,UP],
  HandGripPosition[NEG,ZERO,ZERO,ZERO]],{6}];
(* 5 up *)
Do[SensorControlCycle[SwitchPositions[TRAN,UP],
  HandGripPosition[POS,ZERO,ZERO,ZERO]],{5}];
(* nothing, just keep floating in space *)
Do[SensorControlCycle[SwitchPositions[TRAN,UP],
  HandGripPosition[ZERO,ZERO,ZERO,ZERO]],{6}];
(* finally, 2 down *)
Do[SensorControlCycle[SwitchPositions[TRAN,UP],
  HandGripPosition[NEG,ZERO,ZERO,ZERO]],{2}];

```

Figure 9. The commands to create the sample trajectory.

hand grip set to forward acceleration. Just before cycle 4 is initiated, thruster 6-F2 breaks, which would be used in this acceleration. This leaves thruster 7-F3 applying an additional torque to the system, which results in an increasing angular velocity. In cycles 9 and 10 the astronaut initiates AAH, but keeps the forward acceleration (cycles 10 to 17 and 20 to 25). AAH is now only able to compensate the additional torque, but not to reduce the angular velocity. Only when the forward acceleration is turned off (cycles 17 to 20 and 25 to 30), AAH shows effect.

The functionality of AAH could be improved by immediately excluding thruster 7-F3 from the translational commands when thruster 6-F2 fails (and thus allowing thruster 3-B3 to be used by AAH instead of 6-F2). This would require a slightly modified and more complex thruster selection logic, providing a higher level of safety for the astronaut.

9 Concluding Remarks

In this article a hybrid model of NASA's SAFER system has been presented using the specification language VDM-SL inside the computer algebra system Mathematica. We demonstrated that the implementation of a VDM-SL package for Mathematica provides both, VDM-SL's powerful language fea-

```

VDMFunction[ ControlCycleTest,
  SwitchPositions × HandGripPosition × RotCommand ->
  ThrusterSet,
  ControlCycleTest[SwitchPositions[mode_, aah_], rawGrip,
  aahCmd]:=
  SelectedThrusters[HCM'GripCommand[rawGrip, mode],
  aahCmd, AAH'ActiveAxes[], AAH'IgnoreHcm[]],
  True,
  card[RESULT] ≤ 4 ∧ ThrusterConsistency[RESULT]
];

VDMFunction[ BigTest,
  {}->(HCM'SwitchPositions × HCM'HandGripPosition ×
  AUXIL'RotCommand -> ThrusterSet),
  BigTest[]:= map[({switch, grip, aahLaw}->
  ControlCycleTest[switch, grip, aahLaw])|
  {switch∈switchPositions, grip∈gripPositions,
  aahLaw∈allRotCommands }
];

VDMFunction[ HugeTest,
  {}->(HCM'SwitchPositions × HCM'HandGripPosition ×
  AUXIL'RotCommand -> ThrusterSet),
  HugeTest[]:= map[({switch, grip, aahLaw}->
  ControlCycleTest[switch, grip, aahLaw])|
  {switch∈switchPositions, grip∈allGripPositions,
  aahLaw∈allRotCommands }
];

```

Figure 10. The testing functions.

tures, like comprehensions, as well as the mathematical power of Mathematica, e.g. solving differential equation systems.

The SAFER example shows the validation possibilities of such a combined tool. Like in [1] the complex discrete model of the control logic can be validated through testing. This is a cheap technique for raising the confidence that the right model has been specified prior to the application of more expensive formal proof techniques.

However, with the right tool, there is no reason why the continuous models of a hybrid system should be excluded from validation. Such a hybrid validation is more suitable for finding unjustified domain assumptions made in the discrete model. We strongly propose such validations, due to the fact that making wrong assumptions is the weak point of formal verification techniques, possibly leading to correct proofs of the wrong model.

Furthermore, we demonstrated that the visualization features of Mathematica provide a convenient way to communicate a model to a customer. Moreover, in contrast to [1], our visualization is a functional graph that facilitates the communication to control experts as well as to customers with a technical expertise.

In the Irish school of VDM, Mathematica has been used to explore explicit VDM specifications [14], but to our present knowledge not for modeling hybrid systems.

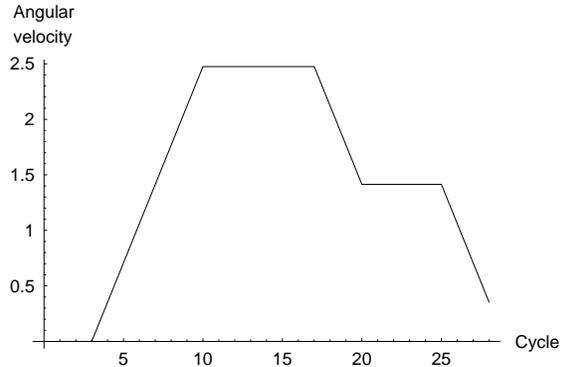


Figure 11. Angular velocity with a broken thruster, AAH initiated in cycle 9.

Note that the conclusion of our work is not that Mathematica is the best tool for validating hybrid system specifications. Our Mathematica approach has its disadvantages, too: Our VDM-SL representation is not as readable as the notation of standard VDM-SL and a typed language would be more suitable for specification purposes. Rather than proposing a certain tool, our work points out the features a powerful toolset should provide for validating hybrid systems.

Another future approach would be the integration of a classic formal method tool with a computer algebra system. For example a combination of Mathematica with the IFAD VDM-SL Toolbox used in [1] would be a possibility. This could be realized with the lately developed CORBA API of this tool, that enables access to the toolbox as a CORBA object and thus calling its VDM-SL interpreter from programs implemented in C or Java. Mathematica provides an interface through its MathLink facility.

Summarizing, we feel that our approach of hybrid validation is a valuable technique for producing systems of higher reliability and hope that it will stimulate further research in this area.

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