H. Föllmer, A. Schied: Stochastic Finance – An Introduction in Discrete Time. de Gruyter Studies in Mathematics 27, Walter de Gruyter, Berlin, New York, 2002, IX+422 pages, Hardcover. ISBN 3-11-017119-8.

This book can be regarded as one of the standard text books of financial mathematics in discrete time. Mathematically, it is more demanding than other introductions, like Pliska's "Introduction to Mathematical Finance", while on the other hand it avoids continuous-time markets and thus also Itô-calculus altogether so that the prerequisites are still relatively low.

The book is structured into two large parts: The one-period model in Part I and dynamic hedging in the more advanced multi-period model in Part II. While the one-period model is very simple, it still displays almost all of the fundamental concepts of the no-arbitrage theory. The multi-period model in Part II on the other hand allows for a restructuring of the portfolio and thus allows one to adjust to newly-available information at time t, modelled by a filtration $\mathcal{F}_t$. In order to replicate a given payoff, this is even required, and the theory of dynamic hedging deals with the question how to restructure the portfolio at each intermediate time.

Part I ("Mathematical finance in one period") starts with a general introduction to the no-arbitrage theory in one time-period, i.e. the market model that is based on the assumption that there should be no possibility of a riskless gain. Martingale measures are introduced and the connection between the existence of martingale measures and the absence of arbitrage is shown, as well as their use for pricing derivative securities. Also, the completeness of a market is discussed and its equivalence to the uniqueness of the risk-neutral measure is shown in the one-period model.

In Chapter 2 follows the introduction of subjective risk evaluation, using utility functions and comparing different portfolios using their expected utilities. In contrast to other introductions to expected utility, this chapter also focuses on uniform preferences, meaning that the particular choice of a utility function is not relevant for an investment strategy to be chosen over another.

Chapter 3 deals with the question of portfolio optimization for a given initial wealth. First, uniqueness of the maximizer is considered and relative entropy methods for special utility functions are presented. The rest of the chapter deals with the classical portfolio optimization methods and with the theory of microeconomic market equilibrium (Arrow-Debreu equilibrium).

The final chapter of the first part, Chapter 4, gives an introduction to risk measures in the one-period model, introducing convex and coherent measures of risk. A large part of this chapter is dedicated to the most common (but unfortunately not coherent) risk measure, Value at Risk, and its generalizations to form coherent risk measures.

Part II ("Dynamic hedging") develops a dynamic version of the market model and the corresponding portfolio theory based on the no-arbitrage condition.

Chapter 5 generalizes the one-period market model of Chapter 1 to the classical discrete-time multi-period model. The structure of this fundamental chapter is straightforward: After a definition of the basic properties like self-financing trading strategies and its consequences, the connection between arbitrage opportunities and the lack of an equivalent martingale measure is shown. Using martingale measures, simple European-type contingent claims (only exercisable at the final time T) can now be introduced and priced using expectation under an equivalent martingale measure. After a quick discussion of market completeness, the Cox-Ross-Rubinstein binomial model is presented as the simplest multi-period market model. Using the proper rescaling and taking limits in the binomial model, the chapter ends with the convergence of the binomial model to the Black-Scholes model in continuous time.

As the multi-period model allows a richer structure of contingent claims than European-type claims, Chapter 6 discusses American-type contingent claims, which the holder can also exercise at certain times before T. Hedging strategies for the seller using Doob's decomposition theorem for the supermartingale price process are discussed as well as stopping strategies for the seller of American-type claims. Using the Snell envelope and stopping times, the optimal exercise time is investigated and arbitrage-free prices are determined using the methods developed in the previous chapters.

The final four chapters of the book discuss the problem of hedging in incomplete markets, where the martingale measure – and thus also the arbitrage-free price – is not unique.

In incomplete markets, a claim can not necessarily be replicated exactly, so Chapter 7 deals with superhedging, which means to replicate a claim that generates at least the required amount in every possible market state. The methods used are again supermartingales in connection with Snell envelopes and the Doob decomposition.

Chapter 8 presents efficient hedging techniques, as superhedging strategies are always on the safe side and thus require unnecessarily high prices. Using various measures of risk, efficient hedging results in hedging strategies that not necessarily generate the required amount in every possible market state, but still with a high probability. This is discussed using the quantile method, which corresponds to the Value at Risk, and the method of minimal shortfall risk.

When the trading of assets and thus the restructuring of the portfolio is not always possible as it was assumed in the frictionless market of the first eight chapters, several convex trading constraints are introduced into the market model. Their effects on the absence of arbitrage are discussed in Chapter 9, as well as superhedging in markets with friction.

The final Chapter 10 takes an alternative approach to hedging in incomplete markets by minimizing the quadratic hedging error. Another approach presented is the variance-optimal hedging method.

In summary, the book is a very good introduction to mathematical finance in the discrete-time setting. The structure of the book follows the well-established pattern of typical financial mathematics courses and books. It is well suited for an advanced introductury lecture on financial mathematics, although the lack of exercises is a significant drawback in that regard. The final chapters deal with more advanced topics that might be treated in a subsequent lecture.

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